

## MATHEMATICAL AND STATISTICAL FOUNDATIONS

Time: 3 Hours

Max. Marks: 70

Question Paper consists of FIVE units, each carrying 14 marks  
 Each unit has TWO questions; either of them should be answered  
 All parts of a question must be answered at one place.

\*\*\*\*\*

## UNIT-I

1. a) State and prove addition theorem on probability for two events. A card is drawn from a well shuffled pack of cards. What is the probability that it is either a spade or an ace? 7M  
 b) A random variable  $X$  has the following probability function: 7M

$X=x$	0	1	2	3	4	5	6	7	8
$P(X=x)$	$a$	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$7a$

Find i)  $a$  ii)  $P(X < 3)$ ,  $P(X \geq 3)$  and  $P(0 < X < 5)$  iii) mean (iv) variance v) distribution function

(OR)

2. a) State and prove Baye's theorem 7M  
 b) In a bolt factory, machines  $A$ ,  $B$  and  $C$  manufacture 20%, 30% and 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from (i) machine  $A$ , (ii) machine  $B$ , and (iii) machine  $C$  7M

## UNIT-II

3. a) A population consists of 5 numbers 2,3,6,8,11. Consider all possible samples of size two which can be drawn with replacement from this population. Find (i) the mean of the population (ii) the standard deviation of the population (iii) the mean of the sampling distribution of means (iv) the standard deviation of the sampling distribution of means. 7M  
 b) An ambulance service claims that is taken on the average less than 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and variance of 16 minutes test the claim at 0.10 level of significance. 7M

(OR)

4. a) The mean and standard deviation of a population are 11,795 and 14,054 respectively. What can one assert that 95% confidence about the maximum error if  $\bar{X} = 11795$  and  $n = 50$ . 7M  
 b) What is the maximum error one can expect to make with probability 0.9 using the mean of a random sample of size  $n=64$  to estimate the mean of a population with  $\sigma^2 = 2.56$  7M

## UNIT-III

5. a) Explain Yates' Correction for Continuity Coefficient of Contingency. 7M  
 b) A company manufactures rope whose breaking strengths have a mean of 300 lb and standard deviation 24 lb. It is believed that by a newly developed process the mean breaking strength can be increased, (a) Design a decision rule for rejecting the old process at a 0.01 level of significance if it is agreed to test 64 ropes, (b) Under the decision rule adopted in (a), what is the probability of accepting the old process when in fact the new process has increased the mean breaking strength to 310 lb? Assume that the standard deviation is still 24 lb. 7M

(OR)

6. a) Two random samples reveal the following results: 7M

Sample	Size	Sample Mean	Sum of the squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples came from the same normal population.

- b) 4 coins were tossed 160 times and the following results were obtained.

7M

Number of heads	0	1	2	3	4
Observed frequency	17	52	54	31	6

Under the assumption that coins are balanced, find the expected frequencies of 0, 1, 2, 3 and 4 heads by using Binomial distribution and test the goodness of fit at 5% level of significance.

#### UNIT-IV

7. a) Show that set  $G$  of all numbers of the form  $a + b^2$ ,  $a, b \in \mathbb{I}$  form a group under the operation  $(a + b^2) + (c + d^2) = (a + c) + (b + d)^2$ . 7M
- b) If  $f: G \rightarrow G$  be a homomorphism with kernel  $K$ , then prove that  $K$  is a normal subgroup of  $G$ , and  $G/K$  is isomorphic to the image of  $f$ . 7M

(OR)

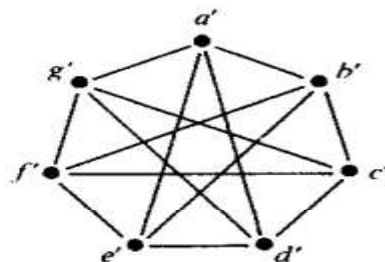
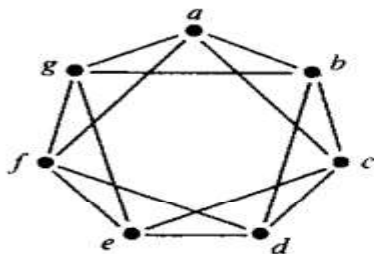
8. a) Prove that every integer  $n > 1$  can be written as a product of primes. 7M
- b) Let  $a = 8316$  and  $b = 10920$ . 7M
- (a) Find  $d = \gcd(a, b)$ , the greatest common divisor of  $a$  and  $b$ .
- (b) Find integers  $m$  and  $n$  such that  $d = ma + nb$ .
- (c) Find  $\text{lcm}(a, b)$ , the least common multiple of  $a$  and  $b$ .

#### UNIT-V

9. a) Draw the graph  $G$  corresponding to the adjacency matrix: 7M

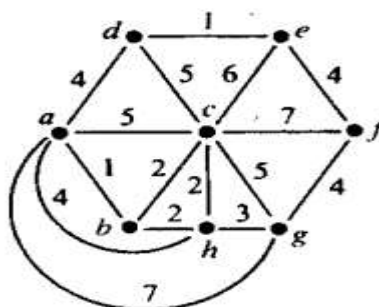
$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- b) Show that the following graph are isomorphic. 7M



(OR)

10. a) Find the minimal spanning tree from the following graph 7M



- b) Find the Chromatic number of the following graph. 7M

